

Physics 681: Solar Physics and Instrumentation – Lecture 13

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Linear Adiabatic Oscillations

- Small disturbances about equilibrium state
- Linearization: Neglect products of two or more small quantities
- Eulerian perturbations (fixed position)
- Lagrangian perturbations (displaced from equilibrium position)
- Adiabatic approximation: neglect any heat generation inside the element and any heat exchange with the surroundings

$$\frac{\delta P}{P_0} = \Gamma_1 \frac{\delta \rho}{\rho_0}$$

- Adiabatic exponent

$$\Gamma_1 = \left(\frac{d \ln P}{d \ln \rho} \right)_s$$

- Adiabatic sound velocity

$$c^2 = \Gamma_1 P_0 / \rho_0$$



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- Non-adiabatic oscillations exist only in the solar atmosphere
- Neglect solar rotation ($\Omega/\omega \approx 10^{-4}$)
- Consider linear oscillations
- Linear equations in inertial reference frame
- Equation of continuity (ξ vectorial distance of a gas parcel from its equilibrium position)

$$\rho_1 + \nabla \cdot (\rho_0 \xi) = 0$$

- Equation of the momentum

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} + \nabla P_1 - \frac{\rho_1}{\rho_0} \nabla P_0 + \rho_0 \Phi_1 = 0$$

- Relation between Lagrangian and Eulerian perturbation

$$\delta f = f_1 + \xi \cdot \nabla f_0$$

- Perturbation of the gravitational potential (Poisson's equation)

$$\Delta \Phi_1 = 4\pi G \rho_1$$



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Spherical Harmonic Representation

- The coefficients of the linear equations are known from the solar equilibrium model
- Vorticity of the perturbation has no vertical component

$$\mathbf{r} \cdot \nabla \times \frac{\partial^2 \xi}{\partial t^2} = 0$$

- In polar coordinates (with spherical harmonics)

$$\partial / \partial t \equiv i\omega \neq 0 \Rightarrow \frac{\partial}{\partial \theta} (\sin \theta \xi_\phi) - \frac{\partial \xi_\theta}{\partial \phi} = 0$$

$$\xi = e^{i\omega t} \left(\xi_r(r), \xi_h(r) \frac{\partial}{\partial \theta}, \frac{\xi_\phi(r)}{\sin \theta} \frac{\partial}{\partial \phi} \right) Y_l^m(\theta, \phi)$$

$$(\rho_1, P_1, \Phi_1) = e^{i\omega t} [\rho_1(r), P_1(r), \Phi_1(r)] Y_l^m(\theta, \phi)$$



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- Recast linear equations in polar coordinates

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \xi_r) - \frac{\xi_r g}{c^2} + \frac{1}{\rho_0} \left(\frac{1}{c^2} - \frac{l(l+1)}{r^2 \omega^2} \right) P_1 - \frac{l(l+1)}{r^2 \omega^2} \Phi_1 = 0$$

$$\frac{1}{\rho_0} \left(\frac{d}{dr} + \frac{g}{c^2} \right) P_1 - (\omega^2 - N^2) \xi_r + \frac{d\Phi_1}{dr} = 0$$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi_1}{dr} \right) - \frac{l(l+1)}{r^2} \Phi_1 - \frac{4\pi G \rho_0}{g} N^2 \xi_r - \frac{4\pi G}{c^2 P_1} = 0$$

- Brunt-Väisälä frequency

$$N^2 = g \left(\frac{1}{\Gamma_1 P_0} \frac{dP_0}{dr} - \frac{1}{\rho_0} \frac{d\rho_0}{dr} \right) \quad \text{with} \quad g = -\frac{1}{\rho_0} \frac{dP_0}{dr}$$

- Determine eigenfrequencies with non-trivial solutions for ω
- Since rotation was neglected, the eigenfrequencies are $(2l+1)$ -fold degenerated



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Cowling Approximation

- Neglect perturbations of the gravitational potential

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \xi_r) - \frac{\xi_r g}{c^2} + \frac{1}{\rho_0} \left(\frac{1}{c^2} - \frac{l(l+1)}{r^2 \omega^2} \right) P_1 = 0$$

$$\frac{1}{\rho_0} \frac{dP_1}{dr} + \frac{g}{\rho_0 c^2} P_1 - (\omega^2 - N^2) \xi_r = 0$$

- For large degree l or high frequencies (corresponding to many nodes n in the radial direction) the approximation is better than 1%



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