

Physics 681: Solar Physics and Instrumentation – Lecture 10

Carsten Denker

NJIT Physics Department
Center for Solar–Terrestrial Research



The Atmosphere

- Radiative Transfer – LTE
 - Equation of Transfer
 - Equilibria
 - Absorption Lines in LTE
- Radiative Transfer – Statistical Equilibrium (SE)
 - Model Assumptions
 - Line Radiation and Einstein Coefficients
 - Continuum Radiation
 - Collisions
 - Source Function
 - Equations of Statistical Equilibrium



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Equation of Transfer

- Optical depth

$$d\tau_v = -\kappa_v \rho dr$$

- Radiative transfer equation

$$\mu \frac{dI_v}{d\tau_v} = I_v - S_v$$

- Formal solution

$$I_v(\tau_v, \mu) = I_v(\tau_{0v}, \mu) \exp[-(\tau_{0v} - \tau_v)/\mu] \\ + \frac{1}{\mu} \int_{\tau_v}^{\tau_{0v}} S(\tau'_v) \exp[-(\tau'_v - \tau_v)/\mu] d\tau'_v$$

- Total emergent intensity

$$I_v(0, \mu) = \frac{1}{\mu} \int_0^{\infty} S_v(\tau_v, \exp(-\tau_v/\mu)) d\tau_v$$

- How can we derive the source function from the absolute intensity? Inversion!



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Equilibria

- Thermodynamic equilibrium

- A single temperature T describes the state of the atmosphere everywhere
- Maxwellian velocity distribution
- Ionization and excitation according to the Saha and Boltzmann equations
- Homogeneous and isotropic black-body radiation field
- No temperature gradient!

- Local thermodynamic equilibrium (LTE)

- Locally, a single temperature T describes the atmosphere
- Important simplification: $S_v = B_v(T)$
- Thermalization length has to be shorter than the distance over which the temperature changes
- LTE might not apply to all species of particles
- Good approximation for visible and IR continua, line wings, and weak line profiles

- Non-LTE (a single temperature T is insufficient)

- Radiative interactions are too rare
- Thermalization length is too long



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Absorption Lines in LTE

- Continuum and line absorption coefficients

$$d\tau_\nu = d\tau_c + d\tau_l \quad \text{or} \quad d\tau_\nu = (1 - \eta_\nu) d\tau_c$$

- Intensity at disk center

$$I_\nu(0,1) = \int_0^\infty (1 + \eta_\nu) B_\nu \exp\left(-\int_0^\tau (1 + \eta_\nu) d\tau'\right) d\tau$$

- Doppler and collisional line broadening (Voigt profile)

$$\phi(\nu) = \frac{\gamma}{\sqrt{\pi} \Delta\nu_D} \int_{-\infty}^{+\infty} \frac{\exp\left(-(\nu - \nu')^2 / \Delta\nu_D^2\right)}{\left[2\pi(\nu' - \nu_0)\right]^2 + \gamma^2 / 4} d\nu', \quad \text{with}$$

$$y = \frac{(\nu - \nu')}{\Delta\nu_D}, \quad \nu = \frac{\nu - \nu_0}{\Delta\nu_D}, \quad \text{and} \quad a = \frac{\gamma}{4\pi\Delta\nu_D} \Rightarrow$$

$$\phi(\nu) = \frac{1}{\sqrt{\pi} \Delta\nu_D} H(a, \nu) \quad \text{with} \quad H(a, \nu) = \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(\nu - y)^2 + a^2} dy$$



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Radiative Transfer – SE

- Statistical Equilibrium (SE)

- Electrons are still described by a Maxwell distribution (single electron temperature T_e)
- However, population depends on radiative processes

- Einstein coefficients

- Spontaneous emission $n_U A_{UL} \chi(\nu) / 4\pi$

- Induced emission $n_U B_{UL} I_\nu \psi(\nu) / 4\pi$

- Absorption $n_U B_{LU} I_\nu \psi(\nu) / 4\pi$



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Continuum Radiation and Collisions

- Photoionization $n_j \alpha_j(\nu) I_\nu / h\nu$
- Radiative recombination $n_c (\gamma_j(\nu) + \beta_j(\nu) I_\nu) / h\nu$
- Collisional transitions between two bound states

$$C_{UL} = \frac{g_L}{g_U} \exp\left(\frac{E_U - E_L}{kT}\right)$$

- Collisional transition from and to the continuum

$$C_{Cj} = \left(\frac{h^2}{2\pi m_e kT}\right)^{3/2} \frac{n_e g_j}{2u_c} \exp\left(\frac{E_c - E_j}{kT}\right) \cdot C_{jC}$$



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