

WHAT DOES HELIOSEISMOLOGY TELL US ABOUT SOLAR CYCLE RELATED STRUCTURAL CHANGES IN THE SUN?

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ABSTRACT

Solar oscillations frequencies show a distinct change with solar activity. The changes in frequencies can be used to study the time variation of solar structure. We discuss constraints on the changes in solar structure with time as obtained with helioseismic data covering the last six years. The frequency variations appear to be dominated by changes in the near-surface layers rather than by changes in the structure of the deeper layers.

Key words: Sun: oscillations; Sun: interior.

1. INTRODUCTION

The study of the Sun's variability has been of interest to both geophysicists and astrophysicists. For geophysicists the interest lies in studying the effect on earth's climate. For astrophysicists, the Sun is a fairly typical star with the advantage of proximity which allows the detailed study of many phenomena which are important to stellar physics, among them being magnetic activity.

The most important change that takes place in the Sun on a time-scale shorter than the human life-span, and hence, on a time-span that can directly affect us, is the solar magnetic cycle with a periodicity of 11 years. The total energy output of the Sun has been precisely measured for about two decades now. These measurements show that solar irradiance varies with solar cycle, being highest during maximum activity (Willson & Hudson 1988). Short-term changes on days to months are directly related to the evolution of active regions (Fröhlich & Pap 1989). In this increasingly technological age the effect of the solar magnetic cycle is also being felt as the effect of magnetic storms which can overload power-lines. Thus one of the aims of studying solar variability has been to predict solar activity. There is however, no reliable way to predict solar activity since there is as yet no accepted model of the solar dynamo.

Till very recently, the only way to study the solar cycle was to observe changes that take place at the surface and make models which reproduce the surface phenomena. However, the history of studying solar rotation has shown us how misleading just reproducing surface phenomena can be (see, e.g., the review by Libbrecht & Morrow 1991). Helioseismology has given us the means to study what happens inside the Sun. Solar oscillations frequencies are known to change with time, therefore, we now have the ability to see changes taking place within the Sun even though we cannot yet predict them. We can now follow the changes as a function of time and of both local and global magnetic activity. Helioseismic data thus give strong constraints on the physical mechanisms that might be postulated to explain solar variability.

Solar oscillations are thought to be linear and adiabatic (although adiabaticity breaks down at the outermost layers, where the time scale of heat transfer is smaller than the period of the oscillations). Different modes of oscillation are described by three numbers, the radial order n which is usually the number of nodes in the radial direction. The angular eigenfunctions are described by spherical harmonics Y_m^l , where l is the degree and m the azimuthal order, and $-l \leq m \leq l$. The degree l is the number of nodal lines on the surface, while m is the number of nodes along the equator. Details of the properties of solar oscillations may be found in Christensen-Dalsgaard (1998).

The most complete information on the oscillations is in principle contained in the individual frequencies ω_{nlm} or frequency splittings $\omega_{nlm} - \omega_{nl}$ between modes in the same multiplet, ω_{nl} being the average multiplet frequency, which for cases of slow rotation (as is the case for the Sun) depends only on the structure of the Sun. However, it is more usual to parameterize the rotational splittings in terms of splitting coefficients according to the formula

$$\frac{\omega_{nlm}}{2\pi} \equiv \nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{\max}} a_j(n, l) \mathcal{P}_j^{(l)}, \quad (1)$$

where the basis functions are polynomials related to the Clebsch-Gordan coefficients (Ritzwoller & Lively 1991, Schou et al. 1994). Thus often instead of speaking of frequency splittings, one talks of “ a coefficients.” Because of the structure of the oscillation eigenfunctions, the values of odd-order a coefficients are determined by solar rotation, while those of even-order coefficients are determined by a number of reasons, such as, deviations from spherical symmetry, large scale flows, magnetic fields, etc. The information on all the angular dependences of solar structure and dynamics are in the a coefficients. The mean frequency ν_{nl} depends only on the spherically symmetric structure of the Sun.

In this paper we discuss what we have learned about changes that take place in the solar interior because of the solar activity cycle. We discuss pre-SOHO and pre-GONG results in § 2, the new data used in this work are described in § 3, inversion techniques and results are described in § 4, a brief description of changes in solar dynamics is given in § 5, and we state and discuss our conclusions in § 6.

2. PRE-SOHO AND PRE-GONG RESULTS

That solar oscillations frequencies change with time has been known since the last solar cycle. Elsworth et al. (1990) studied low-degree modes ($l = 0, 1, 2, 3$) of the Sun as obtained by the Birmingham Solar Oscillations Network (BiSON). They showed that the frequencies of low degree modes have varied from 1977 to 1988 in a manner that is correlated with solar activity as measured by the sunspot number. The frequency variation in that period had a peak-to-peak amplitude of $0.46 \pm 0.106 \mu\text{Hz}$. They speculated that the change in frequencies could reflect variations in the solar radius or in the sound speed in the Sun, and these in turn might be due to changes in solar temperature and/or magnetic fields. The BiSON results continue to track solar activity.

The first results of solar-cycle related changes in intermediate degree modes were reported by Libbrecht & Woodard (1990). They showed that measurements of solar oscillation frequencies in 1986 and 1988 showed systematic differences of the order of 1 part in 10,000. The authors averaged the frequency differences of modes with degrees of 5 to 50 at given frequency intervals, after correcting for differences in mode inertia, and showed that the averaged frequency difference was just a function of frequency, and it was a very smooth function of frequency. This is a strong indication that the cause of the frequency changes is confined to the near-surface layers of the Sun. Any perturbation that is localized at some radius r_m introduces a perturbation to the frequencies that is proportional to

$$\cos(2\tau_m\omega + \phi), \quad \tau_m = \int_{r_m}^R \frac{dr}{c} \quad (2)$$

(e.g., Gough 1990), where τ_m is the acoustical depth

Table 1. Data sets used

Set #	Start Day	10.7cm Flux ^a
GONG sets		
1–3	May 7, 1995	75.1
4–6	Aug 23, 1995	73.6
10–12	Mar 26, 1996	72.7
16–18	Oct 28, 1996	73.0
24–26	Aug 12, 1997	91.0
28–30	Jan 3, 1998	106.2
34–36	Aug 7, 1998	131.3
43–45	Jun 27, 1999	163.4
55–57	Sep 1, 2000	167.3
58–60	Dec 18, 2000	166.1
MDI sets		
1216	May 1, 1996	72.4
1432	Dec 3, 1996	73.2
1792	Nov 28, 1997	89.2
1936	Apr 21, 1998	108.5
2224	Feb 2, 1999	130.7
2440	Sep 7, 1999	161.1
2800	Sep 1, 2000	175.3
2944	Jan 23, 2001	164.4
3160	Aug 27, 2001	219.8

^a Units of $10^{-22} J s^{-1} m^{-2} Hz^{-1}$

of the perturbation, c is the sound speed, ω the angular frequency of the oscillation and ϕ a phase. Hence the perturbation contributes to the frequency differences a component which is an oscillatory function of ω with a ‘frequency’ $f = 2\tau_m$ which increases with the depth of the perturbation. Thus provided they have no intrinsic frequency dependence of their own, changes in the near surface layers are expected to introduce frequency shifts that are slowly varying functions of frequency, which is what is seen in the Libbrecht & Woodard (1990) data. The authors also found changes in the even-order a coefficients and speculated that the changes in frequencies and splittings are a result of latitude-dependent changes in the structure of the sun which are correlated with the Sun’s magnetic-activity cycle.

The Mark-I instrument at the Observatorio del Teide have yielded low-degree frequencies for a period of fifteen years between 1984 and 1999. These data too show that the frequencies track solar activity (Jimenez-Reyes et al. 2001). These data also show a hysteresis like behavior in the frequency differences between the rising and falling phases of Cycle 22 (Jimenez-Reyes et al. 1998).

Although oscillation frequencies of low degree modes span a long time period, the Big Bear data of Libbrecht, Woodard & Kaufman (1990) and Libbrecht & Woodard (1990) are the only widely available intermediate and high degree data set from the Cycle 22.

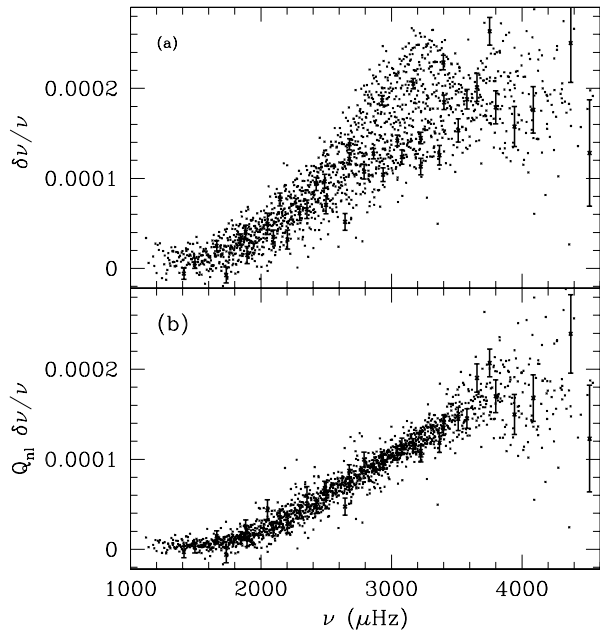


Figure 1. The frequency differences between MDI sets 3160 and 1216 are shown in Panel (a). Panel (b) shows the same frequency differences scaled to eliminate the effects of mode inertia. Only a few error bars are shown for the sake of clarity.

The situation with Cycle 23 is different, with data from both GONG and SOHO, and hence we are now in a position to look in detail at changes that take place within the Sun as the solar cycle progresses.

3. NEW DATA

With the commissioning of the GONG project in 1995 and the helioseismology instruments on board SOHO, such as MDI, GOLF etc., in 1996 we now are able to study the time variation of solar oscillations frequencies in detail. In this work I shall concentrate on results obtained with GONG and MDI. The solar oscillations frequencies from the GONG project were obtained from 108 day time series. The MDI data were obtained from 72-day time series. Some details of how GONG and MDI projects obtain the frequencies are available in Schou et al. (2002). The data sets used in this work are listed in Table 1. The heading ‘‘Start Day’’ is the beginning of the 108 day observational period for the GONG sets and the 72 day period for the MDI set. Also listed in the table is the average 10.7 cm radio frequency flux over the observing period. The 10.7 cm radio frequency flux is known to be a good solar activity index. It should be noted that only a subset of the available GONG and MDI data have been used in this work.

Figure 1 shows the frequency differences between MDI data from late 2001 and mid 1996. Panel (b) of the figure shows the frequency differences corrected

for effect of mode inertia. Modes with low inertia are easier to perturb than modes with high inertia. The quantity Q_{nl} is the ratio of the mode inertia of a mode of degree l order n to that of a mode of degree 0 with the same frequency as the mode of degree l order n (Christensen-Dalsgaard & Berthomieu 1991). Scaling by Q_{nl} eliminates the mode-mass (hence degree) dependence of the frequency differences. We see that the scaled frequency differences seem to be a function only of frequency. To bring out the trends more clearly, we average the scaled frequency differences in groups of 25. These averaged frequency differences are shown in in Fig. 2 where we show different GONG and MDI sets. In the GONG figure we have subtracted out the frequencies of set 10–12 from the other sets. Set 10–12 has the lowest solar activity level among the GONG sets used. In the MDI figure, we subtract out the frequencies of set 1216. Again this set has the lowest activity level.

From Fig. 2 we see that both GONG and MDI data sets behave in similar ways. We also see that the frequency differences for the different sets seem to be a fairly smooth function of frequency. This points to the cause of the differences being fairly close to the surface of the Sun. We invert the frequency differences to check for changes in solar structure.

The change in frequency tracks solar activity very closely. In Fig. 3 we show a plot of the average change in scaled frequency for all MDI modes between 2 mHz and 3 mHz. The change in frequency was calculated with respect to set 1216. Also plotted in the figure is the 10.7cm radio flux, and we can see that the frequency changes and the radio flux, which is an index of solar activity, track each other very closely. This behaviour is the same as that found for low degree modes by the BiSON and Tenerife groups (see § 2).

4. INVERSIONS FOR SOLAR STRUCTURE

4.1. Inversion technique

An inversion for solar structure (e.g. Dziembowski, Pamyatnykh & Sienkiewicz 1990; Däppen et al. 1991; Antia & Basu 1994; Dziembowski et al. 1994) generally proceeds through a linearization of the equations of stellar oscillations around a known reference model using the variational principle. The differences between the structure of the Sun and the reference model are then related to the differences in the frequencies of the Sun and the model by kernels. Non-adiabatic effects and other errors in modeling the surface layers give rise to frequency shifts (Cox & Kidman 1984; Balmforth 1992) which are not accounted for by the variational principle. In the absence of any reliable formulation, these effects have been taken into account in an *ad hoc* manner by including an arbitrary function of frequency in the variational formulation (Dziembowski et al. 1990). Thus the fractional change in frequency of a mode can be

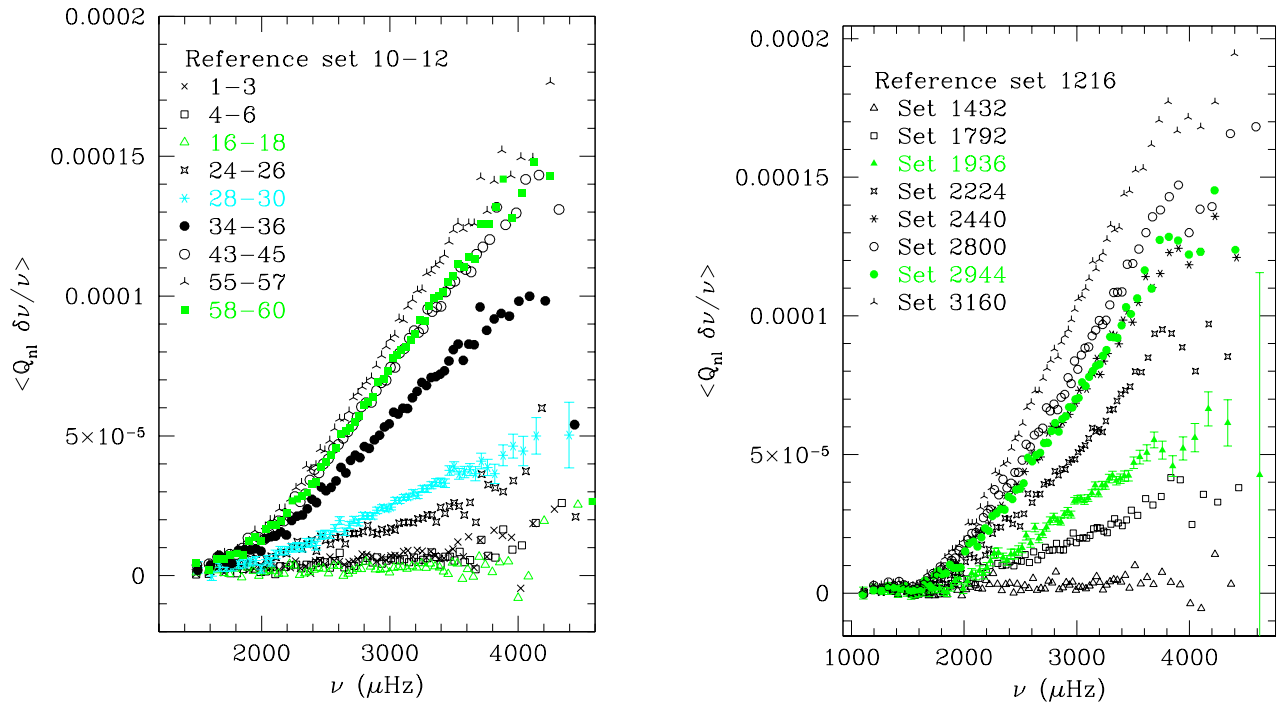


Figure 2. The averaged, scaled frequency differences between different different GONG sets and GONG set 10–12 (left panel), and between different MDI sets and MDI set 1216 (right panel).

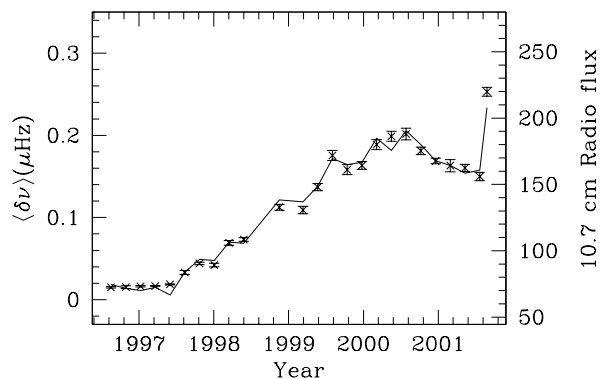


Figure 3. The change in scaled frequencies of different MDI sets plotted as a function of time (continuous line). Also plotted is the 10.7 cm radio flux (points). The frequency differences were taken with respect to set 1216.

expressed in terms of fractional changes in the structure of the model and a “surface term”.

When the oscillation equation is linearized — under the assumption of hydrostatic equilibrium — the fractional change in the frequency can be related to the fractional changes in two of the functions that define the structure of the models. Thus,

$$\frac{\delta\omega_i}{\omega_i} = \int K_{c^2, \rho}^i(r) \frac{\delta c^2(r)}{c^2(r)} dr + \int K_{\rho, c^2}^i(r) \frac{\delta \rho(r)}{\rho(r)} dr$$

$$+ \frac{F_{\text{surf}}(\omega_i)}{E_i} \quad (3)$$

(cf, Dziembowski et al. 1990). Here $\delta\omega_i$ is the difference in the frequency ω_i of the i th mode between the solar data and a reference model. The function $c^2(r)$ and $\rho(r)$ are the sound speed and density profiles. The kernels $K_{c^2, \rho}^i$ and K_{ρ, c^2}^i are known functions of the reference model which relate the changes in frequency to the changes in c^2 and ρ respectively; and E_i is the inertia of the mode, normalized by the photospheric amplitude of the displacement. The term F_{surf} , is the “surface term”, and results from the near-surface errors. The term F_{surf} also contains information of near surface layers that cannot be resolved with the data being inverted.

Equation (3) constitutes the inverse problem that must be solved to infer the differences in structure between the Sun and the reference model. The inverse problems can be solved by a number of different techniques, such as a Regularized Least Squares (RLS) method (see Antia & Basu 1994 for an implementation of the method), or the method of constructing Subtractive Optimally Localized Averages (SOLA) (see Basu et al. 1996; Rabello-Soares et al. 1999 for implementation). All the results in this work were obtained with the SOLA method. The different inversion procedures have been tested thoroughly. In particular, possible errors introduced in the results due to linearization have been studied and found to be negligible (Basu et al. 2000).

For all inversions, we have used Model S of Christensen-Dalsgaard et al. (1996) as our reference

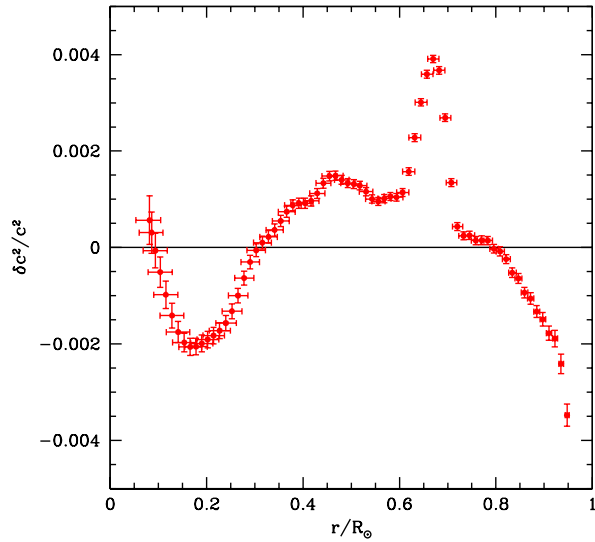


Figure 4. The differences in squared sound speed between the Sun and the reference model.

model. To give a flavour of what the inverted sound-speed difference results look like, we show in Fig. 4 the sound-speed differences between the Sun and the reference model Model S as obtained by inverting MDI frequencies obtained from the first 360 days of observations. The figure shows that in most parts of the solar interior the model is very similar to the real Sun, the sound-speed differences being of the order of 0.1%. However, there is a noticeable difference just below the base of the convection zone (hence forth CZ, which is at approximately $0.71R_\odot$). The solar sound speed is higher in this region. This feature is believed to be caused by an excess of helium below the base of the CZ in the model. Helium diffuses below the CZ base because of gravitational settling. Standard solar models do not have any mixing mechanism below the CZ base hence helium builds up, increasing the mean molecular weight and hence decreasing the sound speed. The base of the CZ in the Sun is however a region of large shear, this is the location of the tachocline. The shear mixes the material below the CZ base and hence the decrease in the solar sound speed is less than the decrease in the sound speed of the model.

4.2. Inversion results

We are mainly interested in seeing how the results shown in Fig. 4 change with time. Since the inversion process is linear, instead of inverting the frequencies of each set and subtracting the inverted results to find the difference, we invert the frequency differences. If there are no changes in solar sound speed, the inversion results should be zero, within limits of error in the inversion results due to errors in the frequencies. There seems to be some systematic differences in the high degree modes of the pre- and post-recovery MDI sets. Since we are not yet

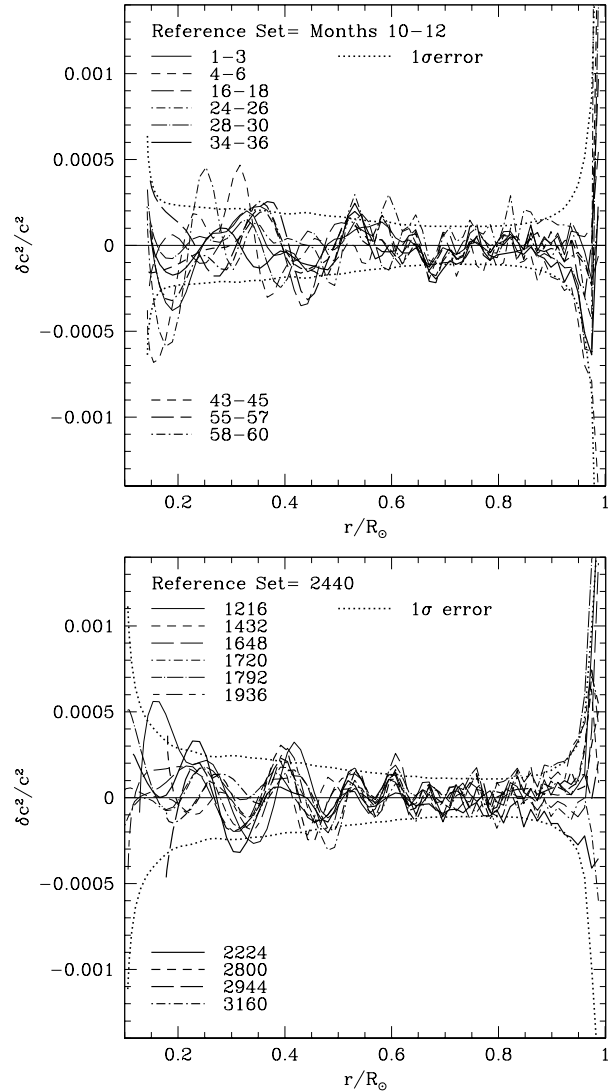


Figure 5. The difference in sound-speed obtained by inverting the GONG frequency changes (upper panel) and MDI frequency differences (lower panel).

sure whether the differences are real or are due to errors, we truncate the MDI mode set above $l = 120$ in order to carry out the inversions. For both GONG and MDI data, we restrict the inverted mode set to modes with frequencies below 3.5 mHz to avoid large surface terms.

We show the sound speed differences obtained by inverting the frequency differences in Fig. 5. These inversion results are reliable from about $0.1R_\odot$ to $0.95R_\odot$. The radius range is imposed by the limitations of the mode sets used in the inversions. We cannot get reliable results at smaller radii due to the lack of sufficient numbers of low degree modes. We cannot get reliable results for radii larger than $0.95R_\odot$ due to lack of high degree modes. We see from Fig. 5 that all the inversion results seem to lie within the 1σ error limits. We do not see any systematic change in the sound speed differences with time at any radius, and hence, conclude that at least within error limits,

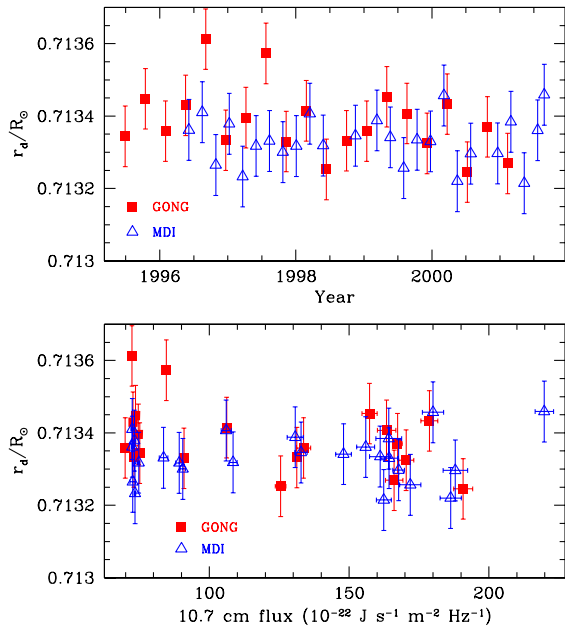


Figure 6. The position of the base of the solar convection zone as a function of time and of the 10.7 cm radio flux.

there is no temporal change in the structure of the Sun.

There is some deviation from the 1σ limit in the results of the outer convection zone, however, we cannot conclude that the changes are due to changes in solar activity. There are two main reasons that can cause spurious deviations from 1σ close to the surface. The first reason is that we have not included high degree modes in the inversions. The MDI sets, as has been discussed earlier have been restricted to modes with $l \leq 120$, the GONG sets are limited to $l \leq 150$. Unfortunately, it is very difficult to determine the frequencies of very high degree modes through global analyses due to the systematic errors involved (e.g., Rhodes et al. 1998). Since inversions at any given radius is most sensitive to modes with their lower turning points at that radius, we are unable to invert the outer regions of the Sun reliably. The second reason is correlation of errors. Even if errors in the input frequencies are uncorrelated, the inversion process correlated the errors in the inversion results at one radius with errors in inversion errors at others (see Howe & Thompson 1996). This problem can be severe at large radii (see Rabello-Soares et al. 1999). This causes the inversion results at large radii to be pulled on one side of the true solution instead of being randomly distributed on both sides of the true solution. The effect is of the order of 1σ which is seen in the inversion results in Fig. 5.

4.3. The base of the convection zone

The seat of the solar dynamo is believed to be close to the base of the convection zone. Eff-Darwich &

Korzennik (2000) in a preliminary analysis of solar cycle related changes in the Sun reported changes at the base of the convection zone. Our inversion results in Fig. 5 fail to show any systematic change at the CZ base. However, there are better ways to look at the CZ base more closely.

On seeing the results of Eff-Darwich & Korzennik (2000), Basu & Antia (2001) investigated the CZ base further, in particular, to see if there is any change in the position of the base of the CZ, which can be determined quite precisely from helioseismology (see Basu & Antia 1997 for details of the method). Basu & Antia (2001) failed to see any change in the position of the CZ base. We have extended the work to newer data sets here. The position of the CZ base as a function of time and solar activity are shown in Fig. 6, and we cannot detect any change in the position of the CZ base either as a function of time or as a function of solar activity. Basu & Antia (2001) did not find any changes in the extent of overshoot below the CZ base either. The lack of change at the base of the CZ is not altogether surprising. If we assume that the changes are caused by magnetic fields, very high magnetic fields to change the structure by adding to the gas pressure of that layer.

4.4. The latitudinal distribution of sound speed

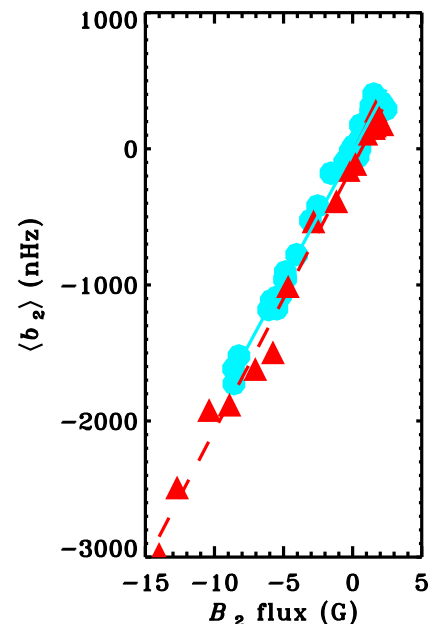


Figure 7. The average a_2 coefficients of different data sets plotted as a function of the corresponding Legendre component of the solar surface magnetic field. Both GONG (triangles) and MDI(circles) data are shown.

Howe, Komm and Hill (1999) have found linear relations between the even-order a coefficients and the

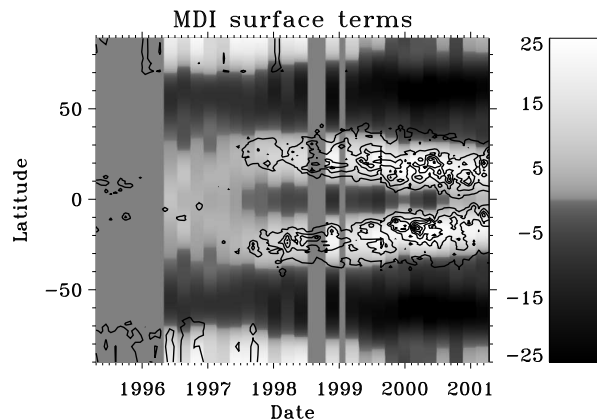


Figure 8. Grey scale map showing the reconstruction of the latitudinal dependence of the surface term from MDI, multiplied by 100. Overlaid contours show the Kitt Peak unsigned magnetic flux with the B_0 term subtracted; contour spacing is 10G

Legendre decomposition components of the surface magnetic flux up to a_8 , and showed that these relations could be extended backwards in time to the BBSO data from the previous cycle. This can be seen in Fig. 7 for the a_2 coefficient. The coefficients have been averaged as per Eq. (5) of Antia et al. (2001), which takes into account the effect of mode inertia. Antia et al. (2001) that a similar relation holds true for all even-order coefficients up to a_{14} . Coefficients of orders higher than 14 were not studied.

Inversions for the latitudinal distribution of sound speed show no changes with time (Antia et al. 2001). This leads to the question again whether all the changes in the even a coefficients are confined to changes in the surface term. This indeed seems to be the case. A plot of the surface term obtained from the inversions is shown in Fig. 8. Superposed on the plot is the surface magnetic field and we can see a clear correlation between the two. Thus as in the case of the frequencies, we are forced to conclude that changes in the even-order a coefficients are caused by changes in the near-surface regions of the Sun.

5. CHANGES IN SOLAR DYNAMICS

Although all changes in solar structure seem to be confined to the outer layers which we are as yet unable to resolve in inversions, changes in solar dynamics can be detected fairly easily.

Solar zonal flows show a change in with time (see Schou 1999, Howe et al. 2000a, Antia & Basu 2000, Antia & Basu 2001, Basu & Antia, this volume). The different low- and intermediate-latitude bands migrate towards the equator, while high latitude bands migrate towards the poles (Antia & Basu 2001, Basu & Antia, this volume). Antia & Basu (2000) found

that these flows penetrate to about $0.1R_\odot$, similar results are found by Howe et al. (2000a).

Results of changes deeper in the Sun are more controversial. Howe et al. (2000b) claim to see a 1.3 year oscillation in the solar rotation rate at the base of the CZ, which Antia & Basu (2000), Corbard et al (2001) and Basu & Antia (this volume) fail to find.

Solar meridional flows change too. The maximum speed of the flow shows a decrease with increase in activity (Basu & Antia, this volume). In addition there are more complex changes on short time-scales (Haber et al. 2001).

6. CONCLUSIONS

Newer more precise solar oscillation frequencies have confirmed that these frequencies change with time. The frequencies show an increase with solar activity and changes are found to be remarkably well correlated with solar activity indices. The change in frequency is almost purely a function of the frequency of the mode, once mode inertia effects are taken into account. This points to changes confined to the near-surface layers of the Sun as being the cause of the observed changes in oscillation frequencies. We have only looked at low and intermediate degree modes in this work. High degree modes obtained from local area analysis (See Bogart et al. this volume) show that changes in the frequencies of high degree modes in active regions are not merely functions of frequency, indicating the possibility of structural changes in the solar interior, in particular in the near-surface regions that are not resolved properly in current inversions. Frequency splittings also change with activity. In particular, the even order splittings are almost linearly correlated with the corresponding component of the solar magnetic field.

Results obtained by inverting the frequencies obtained at different epochs show no significant changes in the structure of the solar interior, at least up to a radius of $0.95R_\odot$. Data sets used in this work do not contain enough high degree modes to allow a reliable inversion of shallower layers. All changes are consistent as being due to errors in the frequencies. We do not see any changes at the base of the solar convection zone.

The latitudinal distribution of solar sound speed does not show any changes with time. The “surface term” is however, highly correlated with the latitudinal distribution of the solar surface magnetic field. This result seems to suggest that the frequencies and splitting coefficients are changed by the direct effects of magnetic fields.

Although solar structure does not appear to show changes with time and activity, solar flows do. Both zonal and meridional flows show changes with time.

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